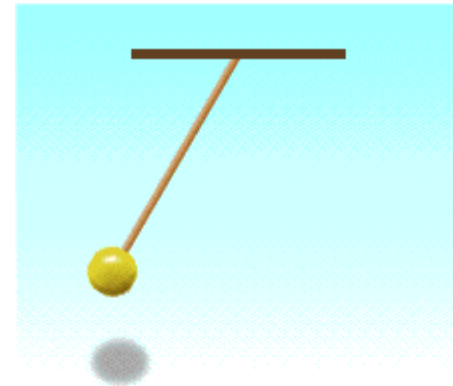# Simple Harmonic Motion
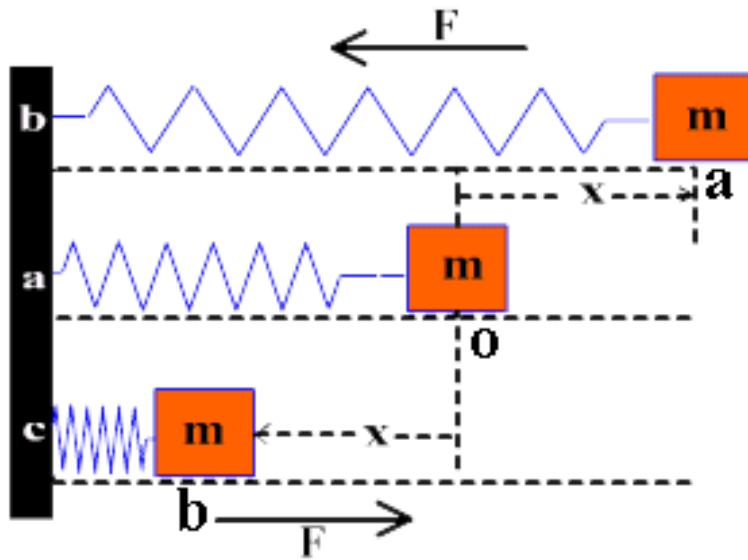
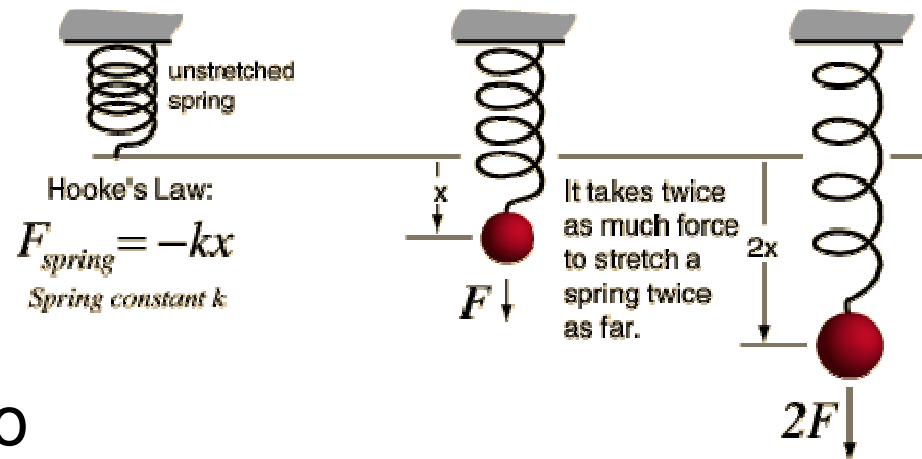AP Physics B

Simple Harmonic Motion

Back and forth motion that is caused by a force that is **directly proportional** to the displacement. The displacement centers around an equilibrium position.

$$F_s \alpha \, x$$

Springs – Hooke's Law

One of the simplest type of simple harmonic motion is called **Hooke's Law.** This is primarily in reference to SPRINGS.

Hooke's Law:

$$F_{spring} = -kx$$

Spring constant k

unstretched spring

It takes twice as much force to stretch a spring twice as far.

x

$F\downarrow$

$2x$

$2F\downarrow$

$F_s \, \alpha \, x$

$k = \text{Constant of Proportionality}$

$k = \text{Spring Constant(Unit : N/m)}$

$F_s = kx \quad or \quad -kx$

The negative sign only tells us that "F" is what is called a RESTORING FORCE, in that it works in the OPPOSITE direction of the displacement.

Hooke's Law

Common formulas which are set equal to Hooke's law are N.S.L. and weight

$$F_s = kx$$

$$F_s = F_g \rightarrow kx = mg$$

$$F_s = F_{net} \rightarrow kx = ma$$

Example

A load of 50 N attached to a spring hanging vertically stretches the spring 5.0 cm. The spring is now placed horizontally on a table and stretched 11.0 cm. What force is required to stretch the spring this amount?

$$F_s = kx$$
$$50 = k(0.05)$$
$$k = \text{1000 N/m}$$

$$F_s = kx$$
$$F_s = (1000)(0.11)$$
$$F_s = \quad \text{110 N}$$
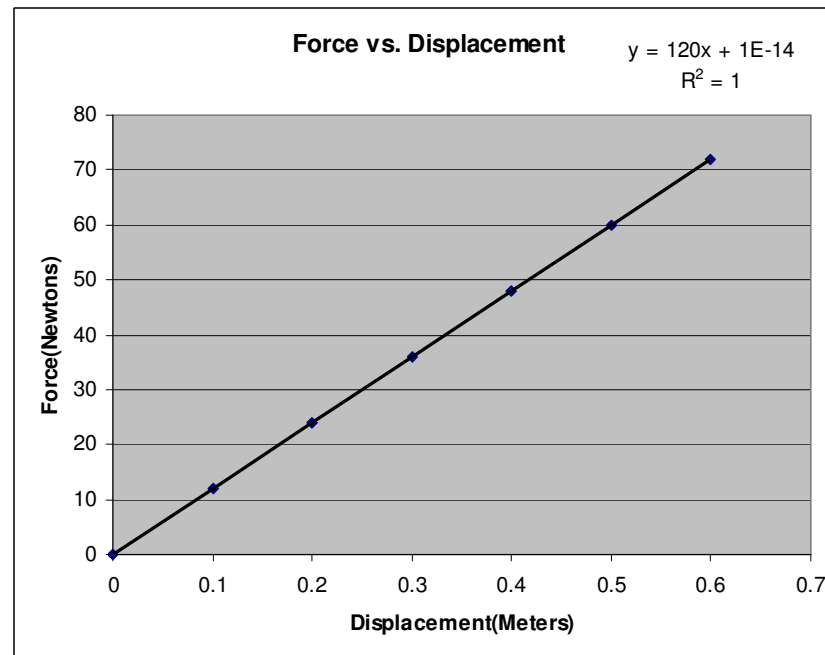
Hooke's Law from a Graphical Point of View

Suppose we had the following data:

x(m)	Force(N)
0	0
0.1	12
0.2	24
0.3	36
0.4	48
0.5	60
0.6	72

$$F_s = kx$$

$$k = \frac{F_s}{x}$$

$$k = \text{Slope of a F vs. x graph}$$

Force vs. Displacement

$y = 120x + 1E\text{-}14$
$R^2 = 1$

k =120 N/m

We have seen F vs. x Before!!!!

Force vs. Displacement

$y = 120x + 1E\text{-}14$
$R^2 = 1$

Force(Newtons) vs. Displacement(Meters)

Area = ELASTIC POTENTIAL ENERGY
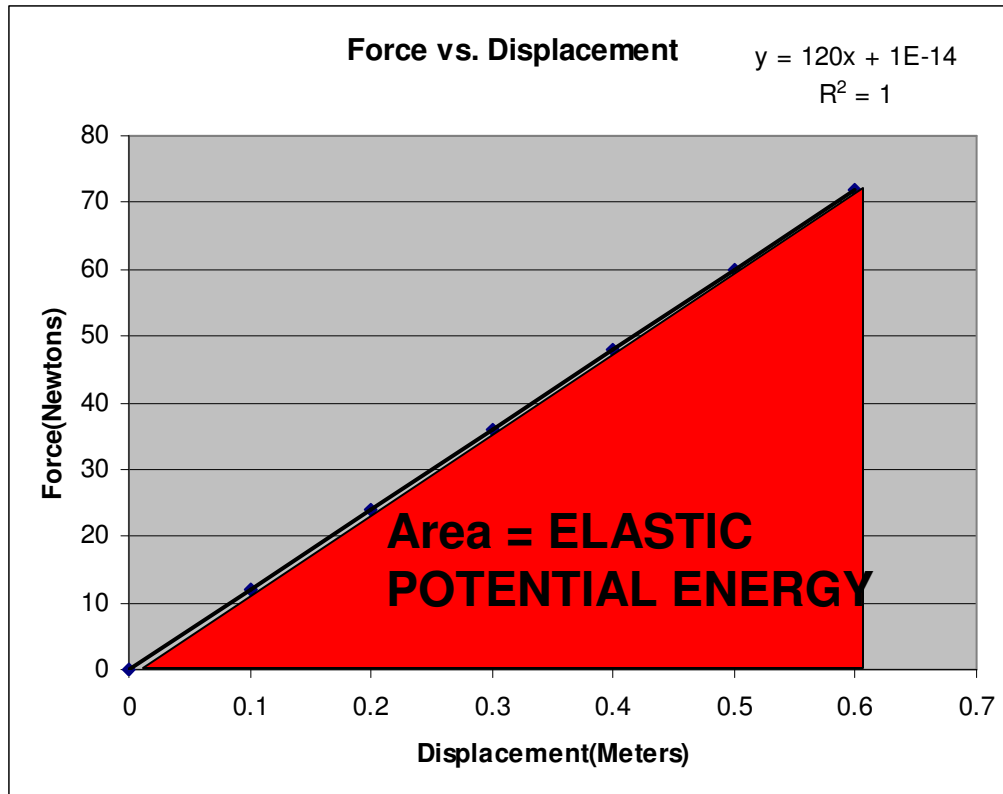
Work or ENERGY = FΔx

Since WORK or ENERGY is the AREA, we must get some type of energy when we compress or elongate the spring. This energy is the AREA under the line!

Since we STORE energy when the spring is compressed and elongated it classifies itself as a "type" of POTENTIAL ENERGY, U_s. In this case, it is called ELASTIC POTENTIAL ENERGY.

Elastic Potential Energy

The graph of F vs.x for a spring that is IDEAL in nature will always produce a line with a positive linear slope. Thus the area under the line will always be represented as a triangle.
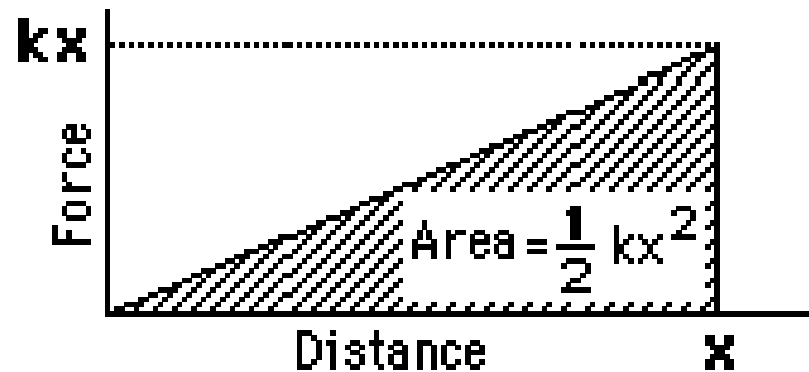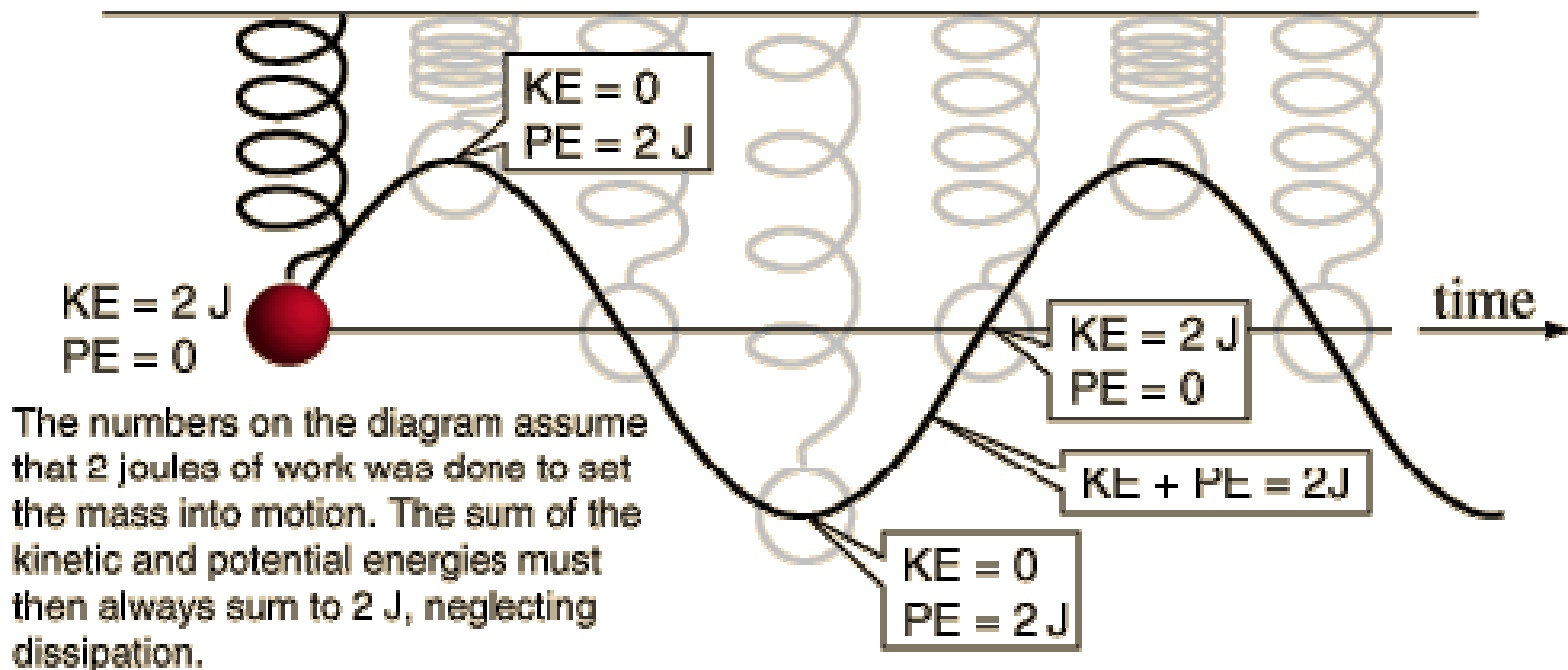

$$U_s = \frac{1}{2}kx^2$$

NOTE: Keep in mind that this can be applied to **WORK** or can be conserved with any other type of energy.

Conservation of Energy in Springs

KE = 0
PE = 2 J

KE = 2 J
PE = 0

The numbers on the diagram assume that 2 joules of work was done to set the mass into motion. The sum of the kinetic and potential energies must then always sum to 2 J, neglecting dissipation.

time

KE = 2 J
PE = 0

KE + PE = 2J

KE = 0
PE = 2 J

Example

A slingshot consists of a light leather cup, containing a stone, that is pulled back against 2 rubber bands. It takes a force of 30 N to stretch the bands 1.0 cm (a) What is the potential energy stored in the bands when a 50.0 g stone is placed in the cup and pulled back 0.20 m from the equilibrium position? (b) With what speed does it leave the slingshot?

$a)\ F_s = kx \quad 30 = k(0.01) \quad k =$ **3000 N/m**

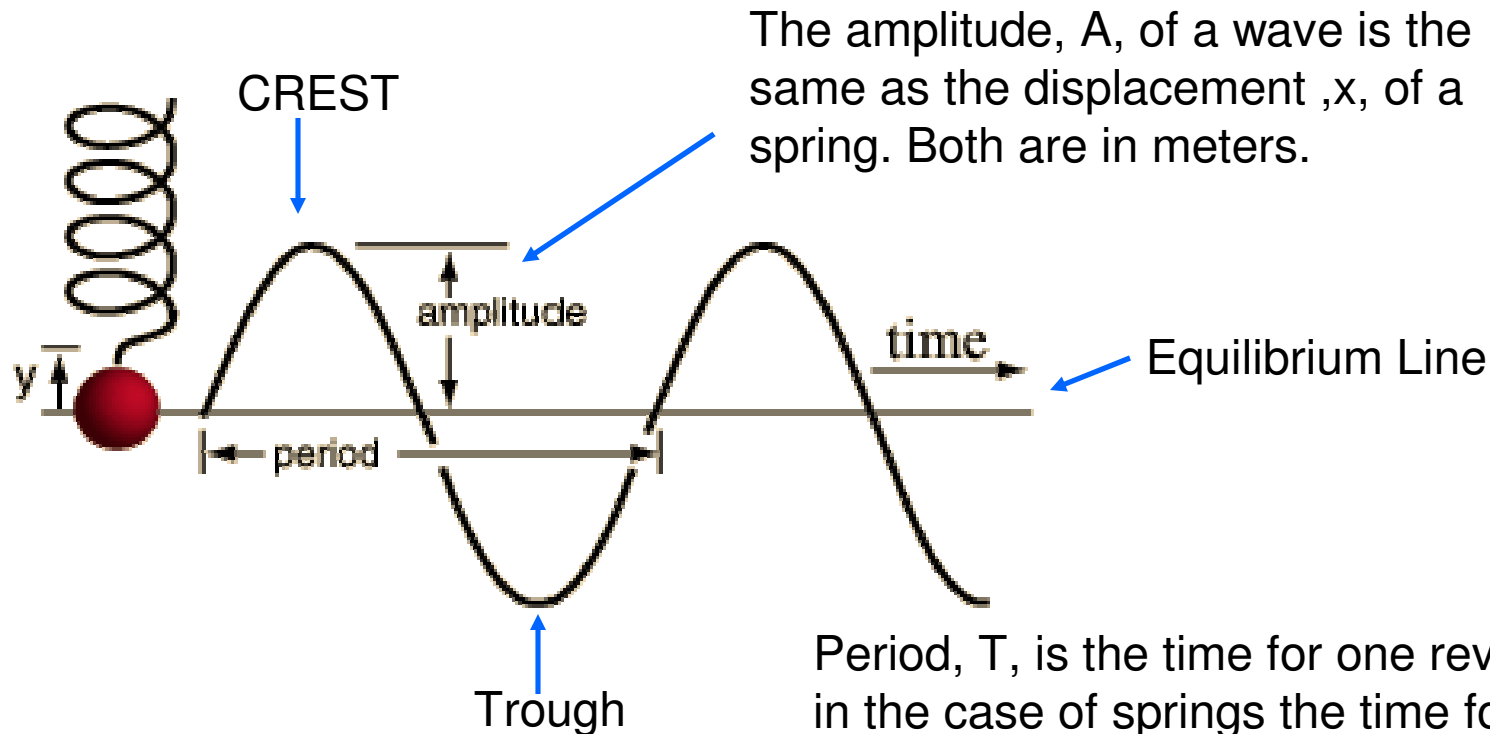$b)\ U_s = \frac{1}{2}kx^2 = 0.5(k)(.20) =$ **300 J**

$c)\ E_B = E_A \quad U_s = K$

$U_s = \frac{1}{2}mv^2 = \frac{1}{2}(0.050)v^2$

$v =$ **109.54 m/s**

Springs are like Waves and Circles

CREST

The amplitude, A, of a wave is the same as the displacement ,x, of a spring. Both are in meters.

amplitude

time

Equilibrium Line

period

Trough

Period, T, is the time for one revolution or in the case of springs the time for ONE COMPLETE oscillation (One crest and trough). Oscillations could also be called vibrations and cycles. In the wave above we have 1.75 cycles or waves or vibrations or oscillations.

T_s=sec/cycle. Let's assume that the wave crosses the equilibrium line in one second intervals. T =3.5 seconds/1.75 cycles. T = 2 sec.

Frequency

The **FREQUENCY** of a wave is the inverse of the PERIOD. That means that the frequency is the #cycles per sec. The commonly used unit is HERTZ(HZ).

$$Period = T = \frac{seconds}{cycles} = \frac{3.5s}{1.75cyc} = 2s$$

$$Frequency = f = \frac{cycles}{seconds} = \frac{1.75cyc}{3.5\sec} = 0.5\frac{c}{s} = 0.5Hz$$
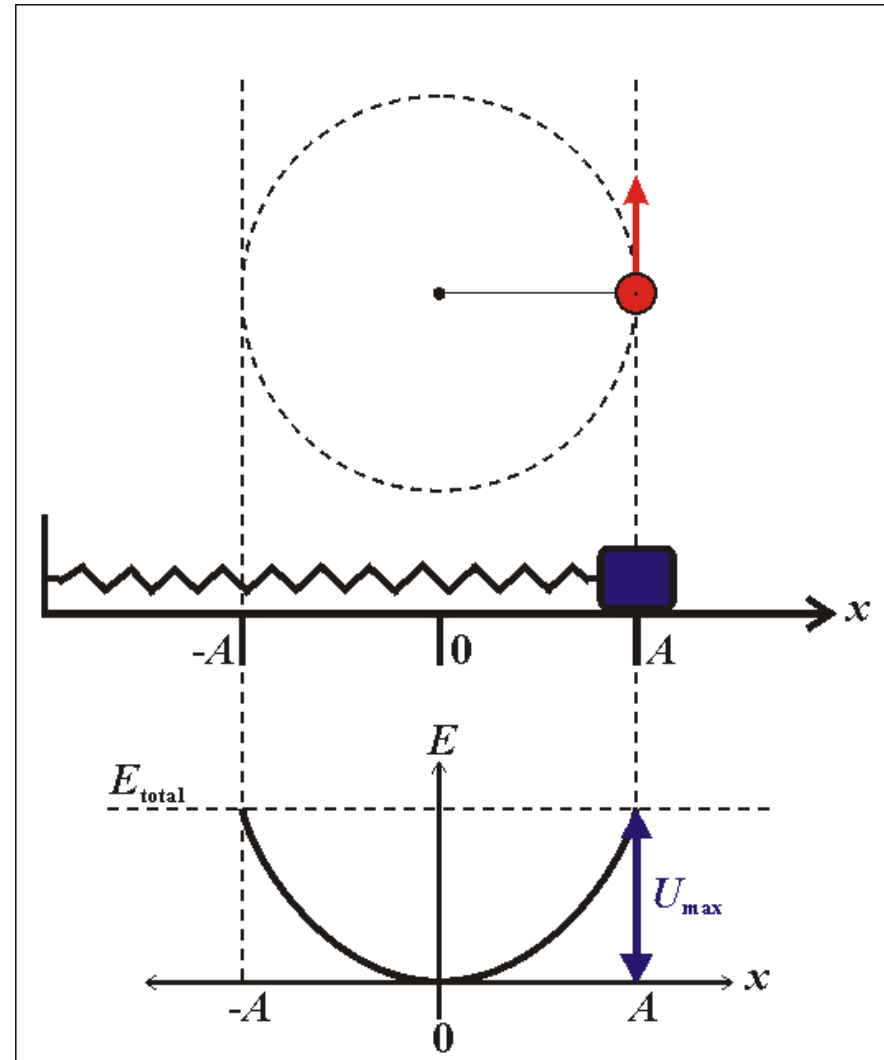
$$T = \frac{1}{f} \quad f = \frac{1}{T}$$

SHM and Uniform Circular Motion

Springs and Waves behave very similar to objects that move in circles.

The radius of the circle is symbolic of the displacement, x, of a spring or the amplitude, A, of a wave.

$$x_{spring} = A_{wave} = r_{circle}$$

SHM and Uniform Circular Motion

$$v = \frac{2\pi r}{T}, \qquad r = A$$

$$v = \frac{2\pi A}{T} \rightarrow \frac{T}{2\pi} = \frac{A}{v}$$

$$Energy_{before} = Energy_{after}$$

$$U_s = K$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \rightarrow kx^2 = mv^2, x = A$$

$$\frac{A^2}{v^2} = \frac{m}{k} \rightarrow \frac{A}{v} = \sqrt{\frac{m}{k}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{k}} \rightarrow T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$

• The radius of a circle is symbolic of the amplitude of a wave.

• Energy is conserved as the elastic potential energy in a spring can be converted into kinetic energy. Once again the displacement of a spring is symbolic of the amplitude of a wave

• Since BOTH algebraic expressions have the ratio of the Amplitude to the velocity we can set them equal to each other.

• This derives the PERIOD of a SPRING.

Example

A 200 g mass is attached to a spring and executes simple harmonic motion with a period of 0.25 s If the total energy of the system is 2.0 J, find the (a) force constant of the spring (b) the amplitude of the motion

$$T_s = 2\pi\sqrt{\frac{m}{k}} \rightarrow 0.25 = 2\pi\sqrt{\frac{0.200}{k}} \qquad k = \quad \textbf{126.3 N/m}$$

$$U_s = \frac{1}{2}kx^2 \rightarrow 2 = \frac{1}{2}kA^2 \qquad A = \quad \textbf{0.18 m}$$
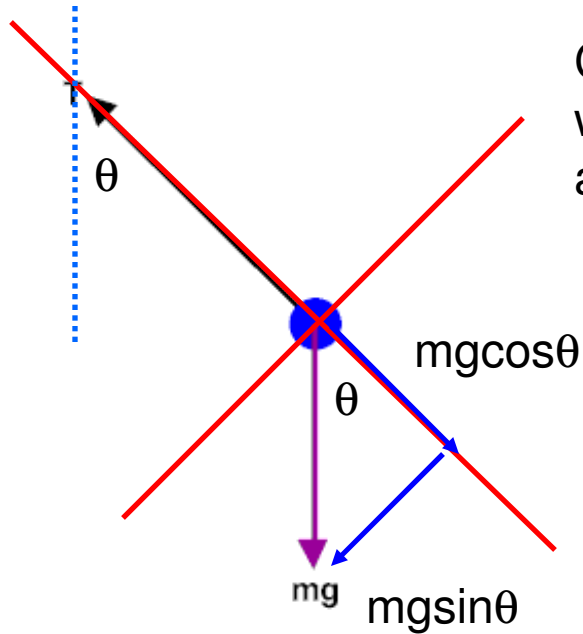
Pendulums

Pendulums, like springs, oscillate back and forth exhibiting simple harmonic behavior.
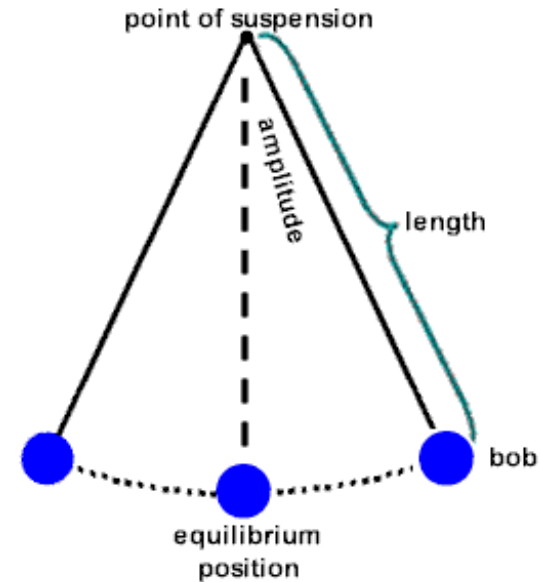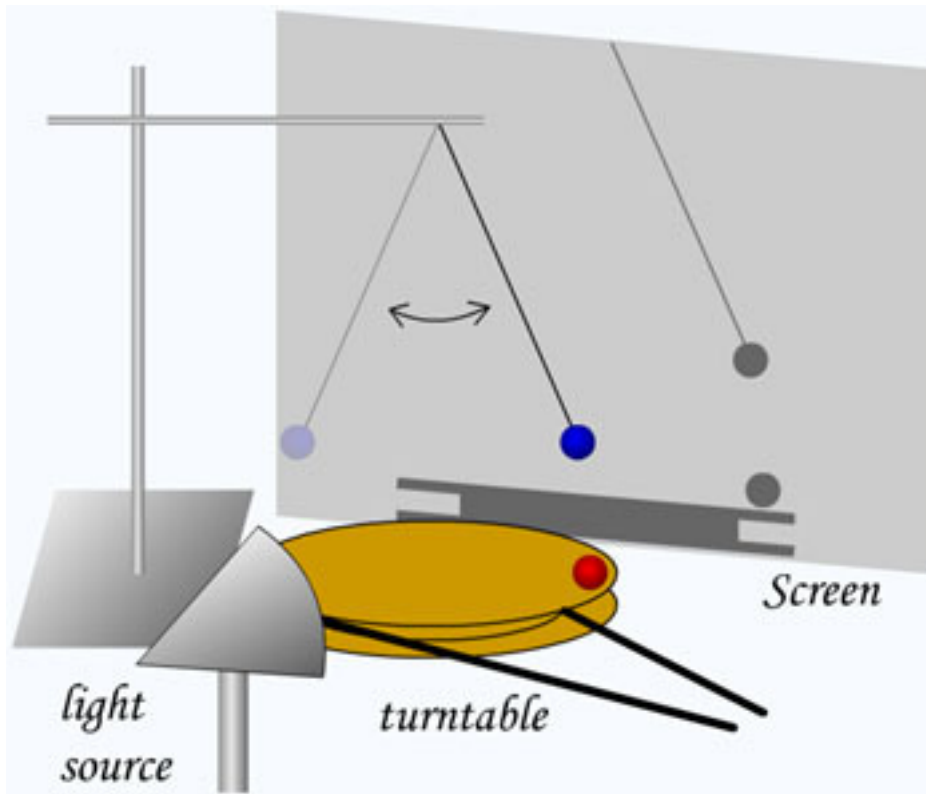
A shadow projector would show a pendulum moving in synchronization with a circle. Here, the angular amplitude is equal to the radius of a circle.

Pendulums

Consider the FBD for a pendulum. Here we have the weight and tension. Even though the weight isn't at an angle let's draw an axis along the tension.

θ

mgcosθ

θ

mg

mgsinθ

$$mg \sin \theta = \text{Restoring Force}$$

$$mg \sin \theta = kx$$

Pendulums

$$mg\sin\theta = \text{Restoring Force}$$

$$\theta = \frac{s}{R} = \frac{s}{L}$$

$$mg\sin\theta = kx$$

$$s = \theta L = Amplitude$$

$$mg\sin\theta = k\theta L$$

What is x? It is the amplitude! In the picture to the left, it represents the chord from where it was released to the bottom of the swing (equilibrium position).

$$\sin\theta \cong \theta, if \ \theta = small$$

$$mg = kl$$

$$\frac{m}{k} = \frac{l}{g}$$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$
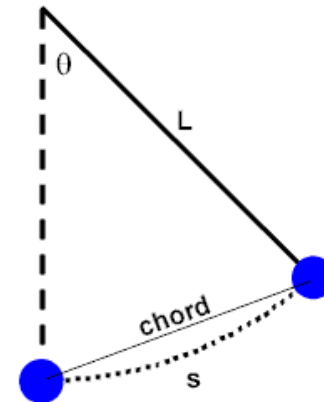
$$T_{pendulum} = 2\pi\sqrt{\frac{l}{g}}$$

Example

A visitor to a lighthouse wishes to determine the height of the tower. She ties a spool of thread to a small rock to make a simple pendulum, which she hangs down the center of a spiral staircase of the tower. The period of oscillation is 9.40 s. What is the height of the tower?

$$T_P = 2\pi\sqrt{\frac{l}{g}} \rightarrow l = height$$

$$T_P^2 = \frac{4\pi^2 l}{g} \rightarrow l = \frac{T_P^2 g}{4\pi^2} = \frac{9.4^2(9.8)}{4(3.141592)^2} =$$

L = Height = 21.93 m